Jet Engine Health Signal Denoising Using Optimally Weighted Recursive Median Filters

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The removal of noise and outliers from health signals is an important problem in jet engine health monitoring. Typically, health signals are time series of damage indicators, which can be sensor measurements or features derived from such measurements. Sharp or sudden changes in health signals can represent abrupt faults and long term deterioration in the system is typical of gradual faults. Simple linear filters tend to smooth out the sharp trend shifts in jet engine signals and are also not good for outlier removal. We propose new optimally designed nonlinear weighted recursive median filters for noise removal from typical health signals of jet engines. Signals for abrupt and gradual faults and with transient data are considered. Numerical results are obtained for a jet engine and show that preprocessing of health signals using the proposed filter significantly removes Gaussian noise and outliers and could therefore greatly improve the accuracy of diagnostic systems. [DOI: 10.1115/1.3200907]

1 Introduction

Health monitoring is a crucial factor in the working and maintenance of jet engines. Due to the harsh environment of their operation and rapidly rotating blades, engines are vulnerable to faults. In order to detect and supervise engine deterioration and faults, certain measurement deviations or deltas are used. To obtain these values, a "good" baseline engine, obtained from a mathematical model, is taken as a reference. Measurements of the "faulty" engine, indicating its condition or health, are obtained from sensors that are strategically placed within the jet engine. Any significant departure from the reference represents the occurrence of a fault. Four typical measurements used in engine health monitoring are exhaust gas temperature (EGT), low rotor speed ($N1$), high rotor speed ($N1$), and fuel flow (WF). These four measurements lead to four measurement deviations or deltas (ΔEGT, $\Delta N1$, $\Delta N2$, and ΔWF). For these gas path measurements, the thrust setting is at the engine pressure ratio. Regular observation of these health signals is vital for the proper functioning of jet engines, since appropriate fault detection can lead to timely corrective measures. However, due to various factors, such as measurement errors and corrupted communication channels, these deltas are often contaminated by noise. For an accurate analysis of the engine's condition, removal of this noise is essential.

Several methods have been proposed to detect faults from an analysis of jet engine health signals. These include the use Kalman filter [1–3], expert systems [4], neural network [4,5], fuzzy logic [6], and probabilistic [7–9] approaches. Comparative studies for neural networks and Kalman filters were also done [10]. Selected studies have addressed the issue of noise removal from signals. DePold and Gass [4] demonstrated the advantages of the exponential average filter, including its faster reaction time to any changes in the measurement signal. The exponential average filter is a simple infinite impulse response (IIR) filter. Typically, the moving average filter is used for denoising gas turbine signals [11]. The moving average filter is a simple finite impulse response (FIR) filter, which is not able to handle rapid changes in the signal. Details about both the FIR and IIR filters and their limitations for gas turbine health signal denoising are discussed in Ref. [11].

Such rapid changes in the measurement signal are often a precursor of a so-called single fault event [10]. Such sudden shifts in signals are typical of abrupt faults and the detection of such faults has been studied by selected researchers for various mechanical systems. For example, Katipamula and Brambley [12] reviewed several works on abrupt faults occurring in building systems. Abrupt changes in strains as an indicator of damage in composite structures were analyzed by Koh et al. [13]. Long term deterioration is typically manifested in a slow change in the measurement signal, which can be well modeled as a linear change. Yen and DeLima [14] studied the fault detection of abrupt and gradual (or incipient) faults in physical plants. In general, health signals comprise mainly of abrupt faults and gradual faults and therefore a general denoising algorithm for such signals should address both the faults simultaneously.

Linear filters such as the FIR and the exponential average can distort the sharp changes in signals and they are also weak at outlier removal [15]. The use of median filters for removal of outliers while preserving sharp trend shifts in signals, which may indicate a fault was used by several researchers. Yeh et al. [16] used median filters for denoising electrocardiography signals, which indicate the heart health of patients. Blanes et al. [17] used median filters for denoising heart rate variability signals. Lee [18] used median filters for filtering measured strain and displacement patterns for improved health monitoring of composite structures. Mba [19] used a median filter for denoising acoustic emission signals for improved bearing health monitoring. One advantage of median type filters is that they remove outliers in signals. Outliers are manifestations of non-Gaussian noise, which can occur in health monitoring systems but is often ignored in the literature. For example, Yoshida [20] pointed out that non-Gaussian noise occurs in health signals because damage tends to be concentrated in a specific part of the structure. He used a Monte Carlo filter to address the issue of non-Gaussian noise in structural damage detection for structures following earthquakes. However, the computer time requirements for such a filtering method can be very large.

We see that median filters can be used to preprocess health signals before subjecting them to fault detection and isolation al-

Manuscript received October 29, 2008; final manuscript received June 19, 2009; published online January 12, 2010. Review conducted by Allan Volponi.

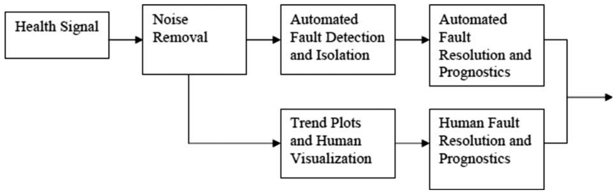

Fig. 1 Schematic representation of health monitoring system

gorithms. There is a possibility of significantly enhancing the median filters for health monitoring preprocessing applications. Progressive improvements in advancing median type algorithms were made over the past decade. Ganguli [15] demonstrated FIR median hybrid (FMH) filters for removing noise from gas turbine measurements while preserving trend shifts. In other works, the use of recursive median (RM) filters [21,22] was put forward and it was found that such filters have excellent noise removal properties.

Several works have addressed denoising preprocessors for better condition monitoring of mechanical systems using other methods. Jafarizadeh et al. [23] pointed out that vibration signals from systems such as gearboxes are noisy and time averaging methods can be used for denoising such signals. They point out that the signal-to-noise ratio (SNR) of such signals is very low and feature extraction of signal components is very difficult. They propose a new noise cancelling method based on time averaging the asynchronous input and use a Morlet wavelet for feature extraction. Xu and Li [24] pointed out that oil monitoring is applied extensively for condition monitoring. However, extraction of features from oil data has always been a problem because of noise contamination. They used a 1D discrete wavelet transform for denoising oil spectrometric data. Abbasion et al. [25] also used wavelet denoising for improving fault classification in roller bearings. Peng and Chu [26] provide a comprehensive review on application of wavelets in condition monitoring and point out their use for denoising signals. Roy and Ganguli [27] compared wavelets with recursive median filters for denoising frequency time series for improved operational health monitoring. They found that while wavelets provide greater levels of noise reduction, recursive median filters provide good results while being much simpler to develop and implement. Moreover, the nonlinear nature of the median type filters makes them useful for the removal of outliers [28,29]

Figure 1 shows a schematic of a health monitoring system. We address the noise removal function in this paper. Noise removal enhances both the automated and human driven actions for health monitoring. In this paper, the weighted recursive median (WRM) filter is introduced for health monitoring applications. The concept of determining the optimal weights for different types of health signals is explored. A comprehensive study of this filter structure shows superior performance compared with the currently used filters. The optimally weighted recursive median filters are tools, which can be of great use for denoising of signals before performing fault detection and isolation functions. The aim of the current work is to develop filters for improved jet engine health monitoring.

1.1 Median Filter. The median filter is a nonlinear digital filtering technique, often used to remove noise from signals. The idea is to examine a sample of the input and decide if it is representative of the signal. This is performed using a window consisting of an odd number of samples. The values in the window are sorted into numerical order and the median value, the sample in the center of the window, is selected as the output. An N-point median filter takes N points surrounding the central point and gives their median as the output, i.e., if x_k represents the input signal, then the output of the median filter [24] is

$$y_k = \text{median}(x_{k-n}, x_{k-n+1}, \ldots, x_k, \ldots, x_{k+n-1}, x_{k+n}) \tag{1}$$

Here, $N=2n+1$ is the window length of the filter. These filters can be effectively implemented for removing non-Gaussian noise, such as outliers, while preserving sharp edges in signals. However, they are not good at removing Gaussian noise. Furthermore, a very large number of iterations can be needed by the median filter to converge. This is because repeated applications of the simple median (SM) filter are needed to remove noise from the data. Moreover, the median filter is noncausal and therefore introduces a time delay in the case of online processing of data obtained from the jet engines.

1.2 RM Filters. A recursive median filter is an advance over the standard median filter. It uses some previous output values for arriving at the next output [25]. This can be expressed as

$$y_k = \text{median}(y_{k-n}, y_{k-n+1}, \ldots, x_k, \ldots, x_{k+n-1}, x_{k+n}) \tag{2}$$

where $N=2n+1$ is the window length of the filter. RM filters provide better immunity to outliers in the data than median filters. Moreover, much lesser iterations are required by these filters to converge. However, RM filters do have certain disadvantages, which limit their efficiency. These filters can introduce blurring effects and can cause "streaking," which is the introduction of steplike artifacts in the filtered signal.

1.3 WRM Filters. The performance of the recursive median filter can be greatly improved by the use of weights. These allow the filter to be tuned to particular types of signals and to reduce the blurring and streaking effects that are observed in the recursive median filter. Moreover, recasting the RM filter in this form provides faster implementation. The weighted recursive median filter [21] can be represented as

$$y_k = \text{median}(w_{k-n} \circ y_{k-n}, w_{k-n+1} \circ y_{k-n+1}, \ldots, w_k \circ x_k, \ldots, w_{k+n-1} \circ x_{k+n-1}, w_{k+n} \circ x_{k+n}) \tag{3}$$

Here $\circ$ stands for duplication and w are the integer weights. Duplication implies that the data point is repeated. For example, $y_k = \text{median}(2 \circ y_{k-1}, 3 \circ x_k, x_{k+1})$ is the same as $y_k = \text{median}(y_{k-1}, y_{k-1}, x_k, x_k, x_k, x_{k+1})$. WRM filters can be of two types based on the weights used: (1) weighted symmetric recursive median filters and (2) weighted asymmetric recursive median filters.

Symmetrically, weighted filters are structures in which the weights are chosen to be symmetric, i.e., $w_{n-i}=w_{n+i}$ [22]. However, in the nonsymmetric structure, the weight values do not follow any particular pattern. Nonsymmetric filters may have advantages over symmetric filters but have not been explored to the best of our knowledge. Signal processing literature proposed adaptive approaches to weighting these filters based on mathematical methods [21]. Such approaches are quite complicated and require a mathematical model of the system.

The primary focus of this paper is to explore the possibilities provided by the weighted filters for noise reduction in health signals. We find the weights which offer the best denoising performance for typical health signals using an optimization approach.

2 Test Signals

A typical jet engine consists of five modules: fan (FAN), low pressure compressor (LPC), high pressure compressor (HPC), low pressure turbine (LPT), and high pressure turbine (HPT), shown schematically in Fig. 2. Air coming into the engine is compressed in the FAN, LPC, and HPC modules combusted in the burner and then expanded through the HPT and LPT modules producing power. The sensors $N1$, $N2$, WF, and EGT provide information about the condition of these modules and are used for health monitoring. In this study, to test the filters, an ideal root signal

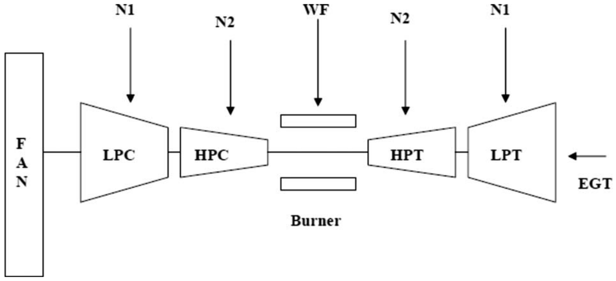

Fig. 2 Schematic representation of jet engine and four basic measurements

ΔEGT with implanted HPC and/or HPT faults is used. The other root signals for ΔN1, ΔN2, and ΔWF can be similarly derived from the engine model [11].

Consider the basic measurement deltas ΔEGT, ΔN1, ΔN2, and ΔWF. In all practical applications, a certain level of noise is always present in the measured signal. As a result, these measurement deltas can be expressed as

$$z = z^0 + \varepsilon \tag{4}$$

where ε represents the noise, z^0 is the pure measurement delta, also called the root signal, and z is the noisy or corrupted signal. Hence, a filter Ψ is required to remove the noise and return the filtered signal for proper damage detection.

$$\hat{z} = \Psi(z) = \Psi(z^0 + \varepsilon) \tag{5}$$

For a comprehensive study of the role of weighted recursive median filters in eliminating noise from jet engine measurements, four different signals are considered. The following signals form the basic representation of the most common types of health signals:

1. step signal (indicating an abrupt fault)
2. ramp signal (indicating a gradual fault)
3. combination signal (comprising both abrupt and gradual faults)
4. transient gas path signal (obtained from Ref. [30])

While the first three signals simulate steady state gas path measurements, the transient type signal can sometime provide information about the engine, which is not true in steady state signals. Each of the first three signals comprises 200 data points. The root signal in Fig. 3 depicts a step signal and it represents a "single fault," which may be triggered by an event such as foreign object damage. Data point $k=60$ represents the onset of this fault. The damage caused is identified as a 2% fall in HPC efficiency and the HPC module is repaired at point $k=140$. In Fig. 4, the development of the HPT fault is illustrated by use of the ramp signal. This

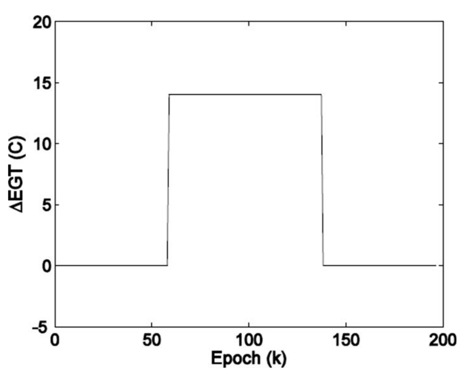

Fig. 3 Step signal representing a HPC fault and its repair

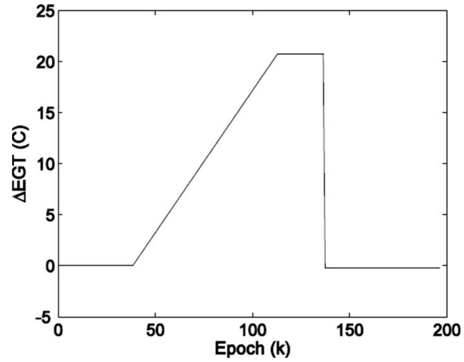

Fig. 4 Ramp signal representing a HPT fault and its repair

fault differs from the HPC one in that it does not occur suddenly and it develops due to engine deterioration. Again, the maximum value of EGT here corresponds to a 2% fall in HPT efficiency. Here, the growth is gradual (approximated by a linear function) from points $k=40-116$. From $k=116$ the HPT fault remains steady and is finally repaired at $k=140$. The step and ramp signals represent the two types of faults considered individually. Now, Fig. 5 shows a combination signal, wherein, both types of faults may occur one after the other. This is a more practical case since any jet engine is susceptible to both these faults. Figure 6 represents a transient signal of a deteriorated engine with a single component fault in the intermediate pressure compressor (IPC) implanted. This signal is obtained from Ref. [30] and is also used in Ref. [31].

In the ideal scenario, the measurement delta is clearly defined at each point. However, to efficiently test the performance of the filters, these signals have been subjected to certain noise levels by

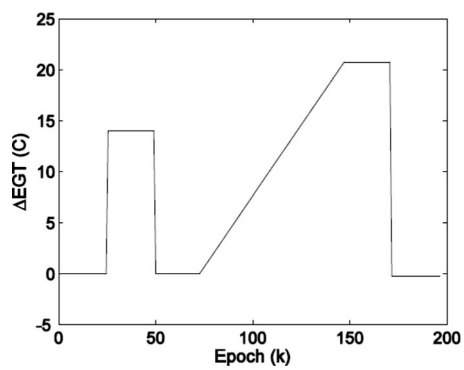

Fig. 5 Combination signal (step and ramp) representing a HPC fault and its repair followed by a HPT fault and its repair

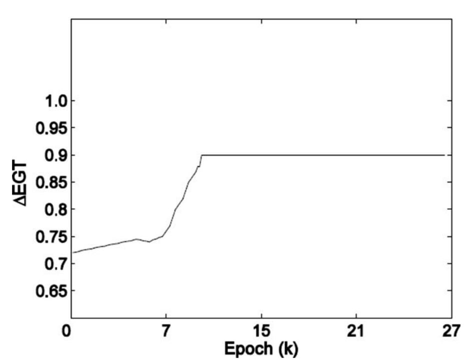

Fig. 6 Transient gas path signal representing IPC fault and transient data

Table 1 Mean rms error estimates of five-point filters on test signals

Signal type	SNR value	SM	RM	WRM
Step	0.1	0.5441	0.4678	0.3806
	0.3	0.5327	0.4532	0.3731
	1.5	0.4620	0.3974	0.3242
Ramp	0.1	0.5602	0.5594	0.4554
	0.3	0.5581	0.5475	0.4481
	1.5	0.4989	0.4889	0.3990
Combination	0.1	0.5971	0.5940	0.4911
	0.3	0.5848	0.5738	0.4826
	1.5	0.5290	0.5134	0.4099
Transient signal	0.1	0.5287	0.4352	0.3446
	0.3	0.5187	0.4243	0.3376
	1.5	0.4509	0.3764	0.2944

using additive white Gaussian noise with varying SNR. Hence, the performance of weighted recursive median filters is studied for high noise (SNR=0.1), medium noise (SNR=0.3), and low noise (SNR=1.5) signals.

3 Numerical Analysis

The weighted RM filter is first compared with the FIR, IIR, and simple median filter, and then to the traditional or unweighted RM filter. The recursive median filter used in this study is a five-point filter with no weights and the weighted recursive median filters used here have been discussed earlier in Eq. (3). To obtain a quantitative idea of the noise reduction, we look at two types of error criteria. The rms error is a measure of the difference between the filtered and the ideal signal. This is given as

$$\text{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\Delta\hat{z}_i - \Delta z_i^0)^2} \tag{6}$$

Here, N is the number of data points in the sample. The minimum absolute error (MAE), which is more sensitive to outliers, is also used to test the filters [21]. We will add outliers to the test signals later in the paper. In the MAE criterion, the error is defined as

$$\text{MAE} = \sum_{i=1}^{N}\frac{1}{N}|\Delta\hat{z}_i - \Delta z_i^0| \tag{7}$$

Tables 1 and 2 summarize the results obtained on passing the test signals through the different filters for the mean rms and MAE estimates, respectively. Since random noise is added to the signals, each noisy signal is different and a large number of such signals should be filtered to arrive at an accurate estimate of the noise reduction given by the filter. Therefore, we use thousand samples of noisy data to arrive at the mean rms and MAE values. We see that the RM filter performs better than the SM filter for all cases. The weighted median filter is discussed next.

Table 2 Mean MAE estimates of five-point filters on test signals

Signal type	SNR value	SM filter	RM filter	WRM filter
Step	0.1	0.4277	0.3576	0.2872
	0.3	0.4190	0.3459	0.2806
	1.5	0.3638	0.3031	0.2428
Ramp	0.1	0.4460	0.4311	0.3506
	0.3	0.4268	0.4210	0.3444
	1.5	0.3856	0.3739	0.3054
Combination	0.1	0.4660	0.4560	0.3790
	0.3	0.4503	0.4403	0.3728
	1.5	0.3999	0.3930	0.3300
Transient signal	0.1	0.4227	0.3454	0.2736
	0.3	0.4156	0.3373	0.2664
	1.5	0.3608	0.2982	0.2328

We now consider the five-point weighted median filter defined as

$$y_k = \text{median}(w_{-2}\circ y_{k-2}, w_{-1}\circ y_{k-1}, w_0\circ x_k, w_1\circ x_{k+1}, w_2\circ x_{k+2}) \tag{8}$$

The filter has five integer weights. The five-point filter keeps the time delay to only two points since x_{k+1} and x_{k+2} is needed by the filter. For many engines, the data are available at a few points during each flight. Therefore, the low filter length keeps the time delay to a minimum while providing sufficient filter length for noise removal, say in comparison to a three-point filter. If data are available more rapidly, longer length filters can be considered.

To obtain the optimal weights we solve the following optimization problem, minimize

$$f(w_{-2}, w_{-1}, w_0, w_1, w_2) = \frac{\sum_{i=1}^{M}\text{rms}}{M} \tag{9}$$

Here, $M=1000$ samples of noisy data are used to obtain a mean rms error and the weights are design variables of the filter, which need to be determined for minimum error. For applications with the weighted filter, all combinations of design variables or weights are computed using integer values {1, 2, 3, and 4}. We found that using higher weights yield the same filter as lower weights because of duplication in the median operation. For example, the weights (4, 1, 3, 2, and 4) give the same result as the weights (8, 2, 6, 4, and 8) in terms of median value. However, the lower weight set is more efficient. Through exhaustive numerical search of the design space, it is observed that several groups of weights could be used to reduce the mean rms error to below that produced by the standard recursive median filter. For computer implementation, the weighted recursive median filter is placed inside a loop of 1000 iterations to obtain the average reduction in noise for a given weight set. This loop in then placed inside a nested loop of depth five, which vary the weights from 1 to 4 in intervals of 1. Thus, the noise removal in terms of rms error for all the integer weights is obtained and the weights corresponding to the minimum values of rms error are selected. Although other optimization approaches such as integer programming or genetic algorithms could be used to generate weights, the exhaustive search guarantees that the best weight set (global minimum) is found.

A similar exercise is performed using the MAE criteria with the objective function, minimize

$$f(w_{-2}, w_{-1}, w_0, w_1, w_2) = \frac{\sum_{i=1}^{M}\text{MAE}}{M} \tag{10}$$

Here, $M=1000$ samples of noisy data are used to obtain a mean MAE and the weights are design variables of the filter, which need to be determined for minimum error. The optimum set of weights is arrived at by determining the lowest rms and MAE error value that could be achieved for each signal. These optimum weights are shown in Table 3 for the rms error and the MAE error. The same set of weights gives both the lowest rms and MAE errors in these cases. The WRM filter results in Table 1 and 2 correspond to the optimal weights in Table 3. We see that the WRM filter shows a significant improvement in noise removal compared with the other filters.

Table 3 Optimal weights for five-point WRM filter using both rms and MAE criteria

Signal type	SNR value	Weights
Step	0.1	[4,1,3,2,4]
	0.3	[4,1,3,2,4]
	1.5	[4,1,3,2,4]
Ramp	0.1	[2,1,2,1,2]
	0.3	[2,1,2,1,2]
	1.5	[2,1,2,1,2]
Combination	0.1	[2,2,2,1,3]
	0.3	[2,2,2,1,3]
	1.5	[2,2,2,1,3]
Transient signal	0.1	[4,1,3,2,4]
	0.3	[4,1,3,2,4]
	1.5	[4,1,3,2,4]

An interesting observation is the lack of one universal sequence of weights that minimizes the error. This implies that there exists an exclusive group of weights for each signal, which can completely minimize errors. Most general steady state signals will be similar to the combination signal and therefore the weight set (2, 2, 2, 1, 3) can be used for such signals. On the other hand, the weights (4, 1, 2, 3, 4) can be used for the transient signals. Physically, the weights mean that certain samples in the signal are given more importance than others. The weights are sensitive to the signal type rather than to noise levels.

Figures 7–10 visually represent the effects of the weighted recursive median filter on the test signals with SNR of 1.5. These

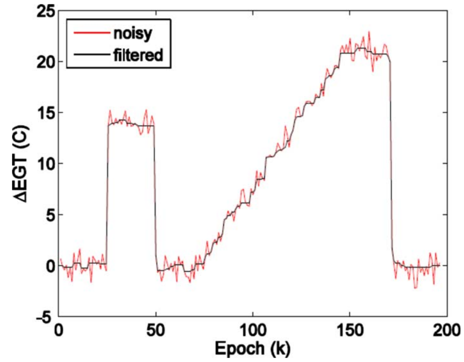

Fig. 9 Effect of weighted RM filters on noisy combination signal with SNR=1.5

figures clearly illustrate the capability of the weighted RM filters to preserve sharp edges or trend shifts in a signal and to remove noise from stationary regions.

4 Test Signal With Outliers

The signal in Fig. 11 considers the combination signal with added noise (SNR=1.5) and outliers. Outliers represent the impulsive noise that may be present in a signal. Here, the outliers are selected at three different levels. The first is equal to 4.23 C and is added at $k=10$, 80, and 140 and subtracted at $k=40$ and 120. The 8.46 C outlier is added at $k=20$, 100, and 190 and subtracted at $k=30$ and 170. The last outlier has a value of 12.69 C and this is added at $k=110$ and 160 and subtracted at $k=60$ and 130. Similarly, outliers are added to the step, ramp, and transient signals. The weights obtained after putting in the outliers are the same as

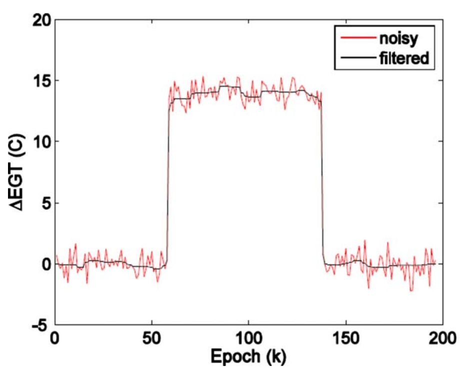

Fig. 7 Effect of weighted RM filters on noisy step signal with SNR=1.5

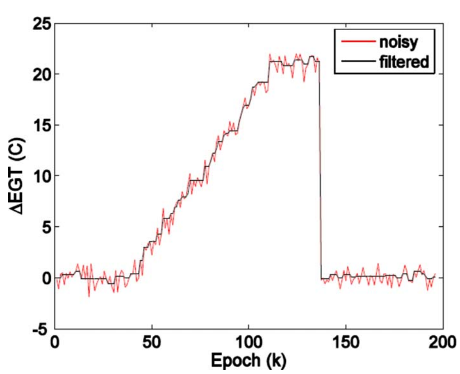

Fig. 8 Effect of weighted RM filters on noisy ramp signal with SNR=1.5

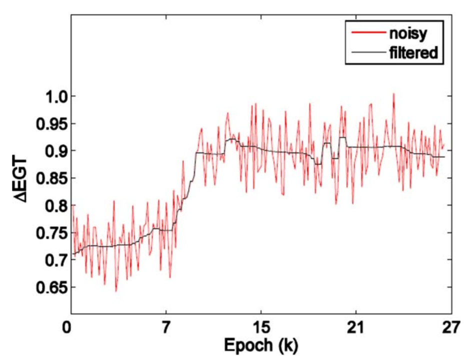

Fig. 10 Effect of weighted RM filters on noisy realistic signal with SNR=1.5

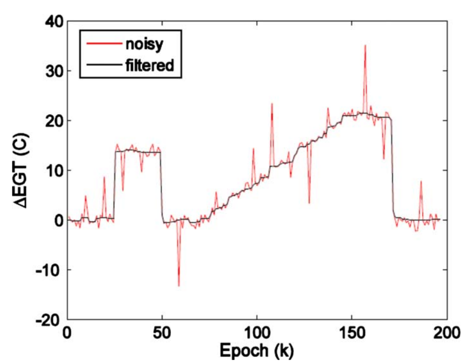

Fig. 11 Effect of weighted RM filters on noisy combination signal with outliers

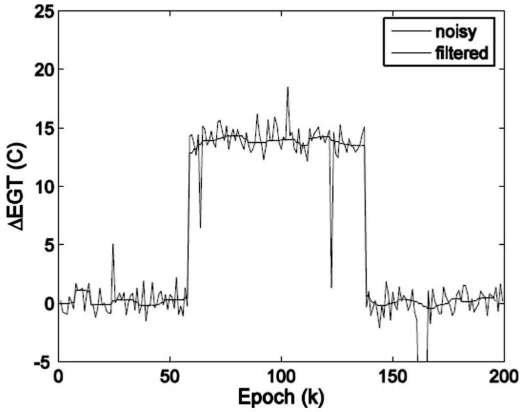

Fig. 12 Effect of weighted RM filters on noisy step signal with outliers

shown in Table 3. This is primarily because all median architectures are good at removing outliers and the weights serve to address the ideal signal characteristics.

The weighted recursive median filter is able to efficiently discard these outliers while preserving signal features, which can be easily observed from Figs. 11–14. On the contrary, results in Tables 4 and 5 show that the simple median and RM filters do not provide the same degree of immunity to noise and outliers. This superior performance of the weighted filter makes it highly suitable for denoising of engine health signals, where the features of the original signal are critical to engine maintenance.

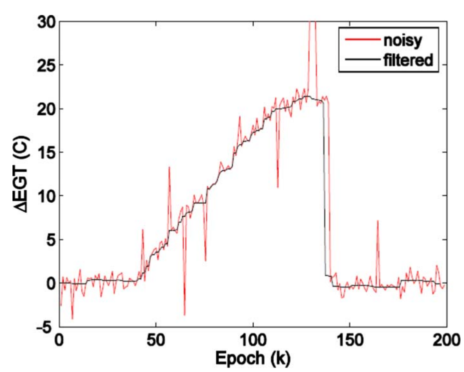

Fig. 13 Effect of weighted RM filters on noisy ramp signal with outliers

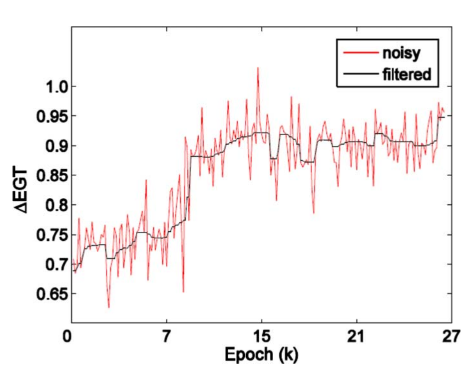

Fig. 14 Effect of weighted RM filters on noisy realistic signal with outliers

Table 4 rms error of different filters on test signal containing outliers

Signal type	SNR value	SM filter	RM filter	WRM filter
Step	0.1	0.5632	0.4819	0.3977
	0.3	0.5523	0.4750	0.3866
	1.5	0.4788	0.4133	0.3424
Ramp	0.1	0.6016	0.5730	0.4922
	0.3	0.5900	0.5646	0.4795
	1.5	0.5272	0.4961	0.4311
Combination	0.1	0.6332	0.6113	0.5291
	0.3	0.6238	0.5994	0.5183
	1.5	0.5500	0.5249	0.4632
Transient signal	0.1	0.5292	0.4372	0.3392
	0.3	0.5182	0.4281	0.3362
	1.5	0.4502	0.3752	0.2968

5 Performance Comparison

This section summarizes the filter performance of the structures used in this study. To get a statistical measure of the effectiveness of each method, 1000 samples of noisy data are generated and filtered as discussed previously. Using the MAE definition, as given in Eq. (7), we can define a parameter for efficiency measurement of these filters in terms of noise reduction as

$$\rho = \frac{\mathrm{MAE}^{(\mathrm{noisy})} - \mathrm{MAE}^{(\mathrm{filtered})}}{\mathrm{MAE}^{(\mathrm{noisy})}} 100 \tag{11}$$

Table 6 clearly illustrates this improvement by comparing the performance of the three filters studied in this work. We observe that the simple median filter provides a noise reduction of only 39–46%; the traditional recursive structure brings about a reduction of 41–56%, while the weighted structure improves this considerably to values ranging from 51% to 65%. This leads to significant accuracy in obtaining the root signal from the contaminated one.

In order to clearly observe the superiority of the weighted filter, we use another parameter η, which is defined as

$$\eta^{(\mathrm{SM})} = \frac{\mathrm{MAE}^{(\mathrm{SM})} - \mathrm{MAE}^{(\mathrm{WRMF})}}{\mathrm{MAE}^{(\mathrm{SM})}} 100 \tag{12}$$

$$\eta^{(\mathrm{RMF})} = \frac{\mathrm{MAE}^{(\mathrm{RMF})} - \mathrm{MAE}^{(\mathrm{WRMF})}}{\mathrm{MAE}^{(\mathrm{RMF})}} 100 \tag{13}$$

Table 7 summarizes the improvement provided by the weighted filters over the simple median and recursive median filters. It is

Table 5 MAE estimate of different filters on test signal containing outliers

Signal type	SNR value	SM filter	RM filter	WRM filter
Step	0.1	0.4422	0.3666	0.2998
	0.3	0.4346	0.3618	0.2918
	1.5	0.3761	0.3156	0.2570
Ramp	0.1	0.4594	0.4511	0.3754
	0.3	0.4495	0.4439	0.3667
	1.5	0.3996	0.3897	0.3275
Combination	0.1	0.4850	0.4791	0.4102
	0.3	0.4774	0.4692	0.4011
	1.5	0.4200	0.4112	0.3576
Transient signal	0.1	0.4249	0.3466	0.2705
	0.3	0.4155	0.3397	0.2671
	1.5	0.3610	0.2967	0.2354

Table 6 Percentage noise reduction provided by different filters for test signals

Signal type	SNR value	$\rho^{(\text{median})}$ (%)	$\rho^{(\text{RMF})}$ (%)	$\rho^{(\text{weighted RMF})}$ (%)
Step	0.1	45.63	54.54	63.49
	0.3	45.54	55.04	63.53
	1.5	45.84	54.90	63.87
Ramp	0.1	43.34	45.23	55.46
	0.3	44.73	45.48	55.40
	1.5	42.64	44.38	54.57
Combination	0.1	40.93	42.20	51.96
	0.3	41.53	42.80	51.62
	1.5	39.06	41.57	50.94
Real signal	0.1	46.55	56.32	65.40
	0.3	46.03	56.20	65.41
	1.5	46.18	55.52	65.27

Table 8 Percentage noise reduction provided by different filters for noisy test signal contaminated with outliers

Signal type	SNR value	$\rho^{(\text{median})}$ (%)	$\rho^{(\text{RMF})}$ (%)	$\rho^{(\text{weighted RMF})}$ (%)
Step	0.1	51.84	60.07	67.35
	0.3	51.90	59.96	67.70
	1.5	53.17	60.70	68.00
Ramp	0.1	57.16	57.93	64.99
	0.3	57.39	57.92	65.24
	1.5	58.24	59.27	65.77
Combination	0.1	62.85	63.30	68.58
	0.3	62.92	63.56	68.85
	1.5	64.83	65.57	70.06
Real signal	0.1	46.16	56.08	65.72
	0.3	46.14	55.97	65.38
	1.5	46.16	55.75	64.89

seen that weighted RM filters improve the effectiveness of the median filters by a maximum of 36% and of the unweighted RM filters by 22%. These results prove the earlier findings that the weighted filter provides several advantages over the standard filters in terms of feature preservation and noise reduction. Note that these advantages are provided by a numerically efficient filter, which is very easy to implement. For the numerical results in this study, the filters were implemented in MATLAB. The results of this paper clearly show that optimization can lead to a better filter for health monitoring systems with negligible increase in complexity and cost.

Tables 8 and 9 show the improvements in performance of the weighted filter over the other filters for signals that are contaminated with Gaussian noise as well as non-Gaussian outliers. From Table 8, we can see that the simple median filter reduces noise by about 46–65%, the recursive median filter by about 56–66%, and the WRM filter by about 65–70%.

For a signal with many outliers, all median type filters work well as they are ideal for outlier removal. Also, the outliers affect the error norms to a much greater extent compared with Gaussian noise. Even then, we see that the WRM filter works well even in the presence of outliers, as can be seen from Table 9.

We have shown that optimally weighted recursive median architectures are a powerful tool for improved as turbine health monitoring. However, there are several aspects about the practical implementation of this method, which can be addressed in future research. An adaptive approach to weight generation based on incoming online data can be explored. Other types of faults with different magnitudes (both bias and slope) should be evaluated. Besides the abrupt faults and gradual faults considered in this work, other faults such as intermittent faults and increase in noise caused by faults are also possible and need to be addressed. We also note that the current approach can be used as a complement and preprocessor to other gas path denoising approaches proposed in literature, which are often tuned to Gaussian noise [32–40].

6 Conclusions

In this paper, a new optimally WRM filter for denoising health signals is proposed. Test signals for abrupt and gradual faults are used for a gas turbine engine diagnostic problem, along with a transient signal. The WRM filter is developed and the weights are optimized for typical health monitoring signals by minimizing the error norms between the noisy and root signal. The WRM filter provides better denoising results compared with the simple median filter and recursive median filter. The WRM filter also improves the visual quality of the signals by removing the noise and outliers while preserving important features of the root signal such as sharp edges and gradual shifts. The WRM filter is presented as a preprocessor for denoising health signals prior to fault detection and isolation in jet engines.

Table 7 Improvement in performance of weighted RM filters over other filters

Signal type	SNR value	$\eta^{(\text{SM})}$ (%)	$\eta^{(\text{RMF})}$ (%)
Step	0.1	32.85	19.69
	0.3	33.03	18.88
	1.5	33.26	19.89
Ramp	0.1	21.39	18.67
	0.3	19.31	18.19
	1.5	20.80	18.32
Combination	0.1	18.67	16.89
	0.3	17.26	15.38
	1.5	19.49	16.03
Transient signal	0.1	35.27	20.29
	0.3	35.90	21.02
	1.5	35.48	21.93

Table 9 Improvement in performance of weighted RM filters over other filters for noisy test signals contaminated with outliers

Signal type	SNR value	$\eta^{(\text{SM})}$ (%)	$\eta^{(\text{RMF})}$ (%)
Step	0.1	32.30	18.22
	0.3	32.86	19.35
	1.5	31.67	18.57
Ramp	0.1	18.28	16.78
	0.3	18.42	17.39
	1.5	18.04	15.96
Combination	0.1	15.42	14.38
	0.3	15.98	14.51
	1.5	14.86	13.04
Real signal	0.1	36.34	21.96
	0.3	35.82	21.37
	1.5	34.79	20.66

Nomenclature

k = discrete time
M = number of points in sample
N = number of noisy samples of data, length of filter
$N1$ = low rotor speed
$N2$ = high rotor speed
w = integer median filter weights
x = input to filter
y = output of the filter
z^0 = ideal measurement delta
z = noisy measurement deltas
$\hat{z}$ = filtered measurement delta
Δ = change from baseline good engine
ε = noise
Ψ = mathematical representation of filter

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